

# Robust Aleatoric Modeling for Future Vehicle Localization

Max Hudnell, True Price, Jan-Michael Frahm  
{mhudnell, jtprice, jmf}@cs.unc.edu

## Overview

**Motivation:** Future object localization is a new area of research with potentially useful application to collision prevention systems and tracking algorithms.

**Goal:** Accurately predict future object localizations (for any given future timepoint) while also modeling their uncertainty.

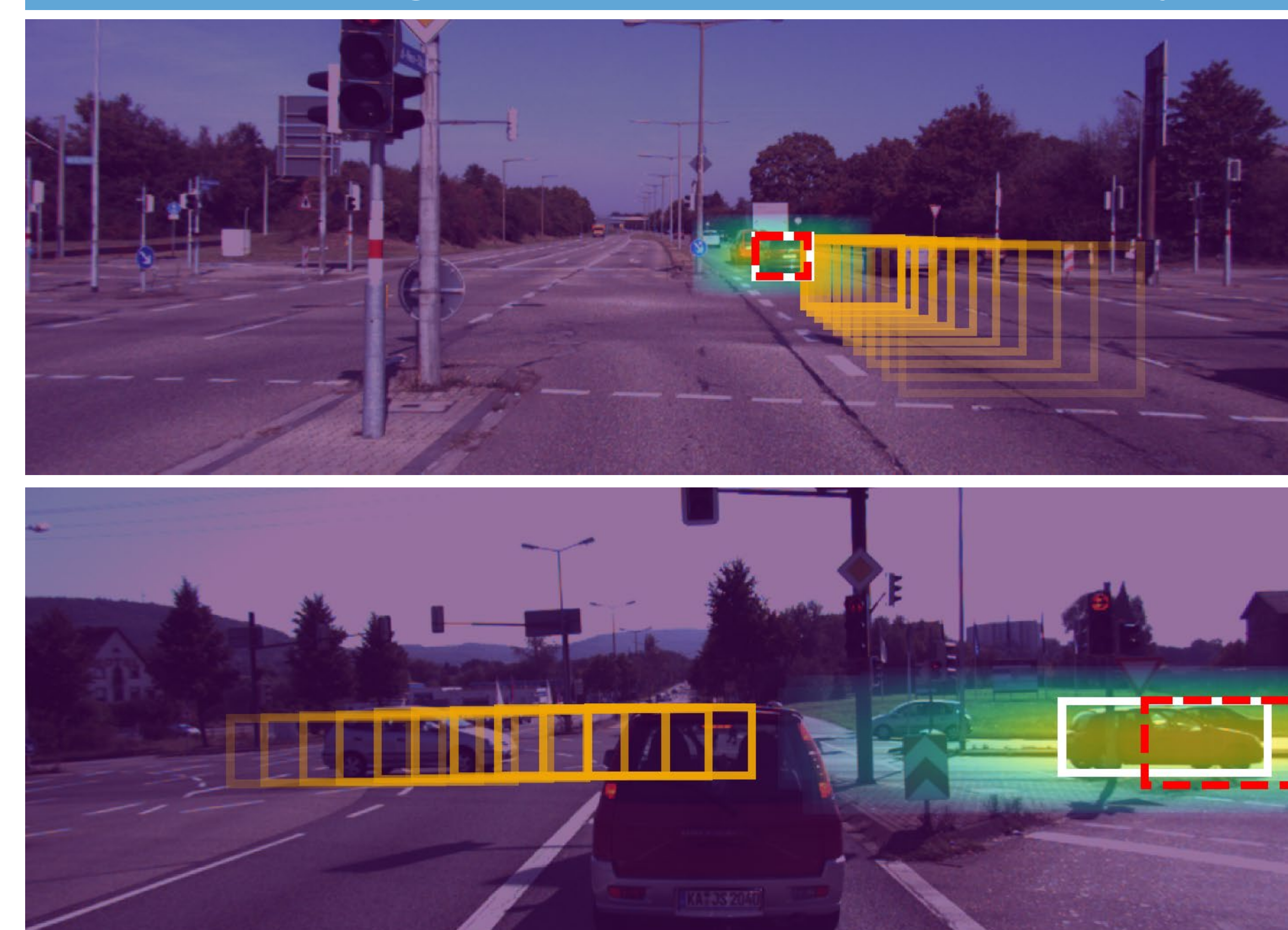
### Challenges:

1. **Overfitting to rare instances** can occur when modeling Gaussian uncertainty.
2. **Current techniques fail to evaluate aleatoric uncertainty** by only evaluating prediction mean

### Contributions:

- We model predictions as a continuous function of time: Less expensive without sacrificing accuracy
- **Robust distribution** formulation
- Introduce technique for **uncertainty evaluation**

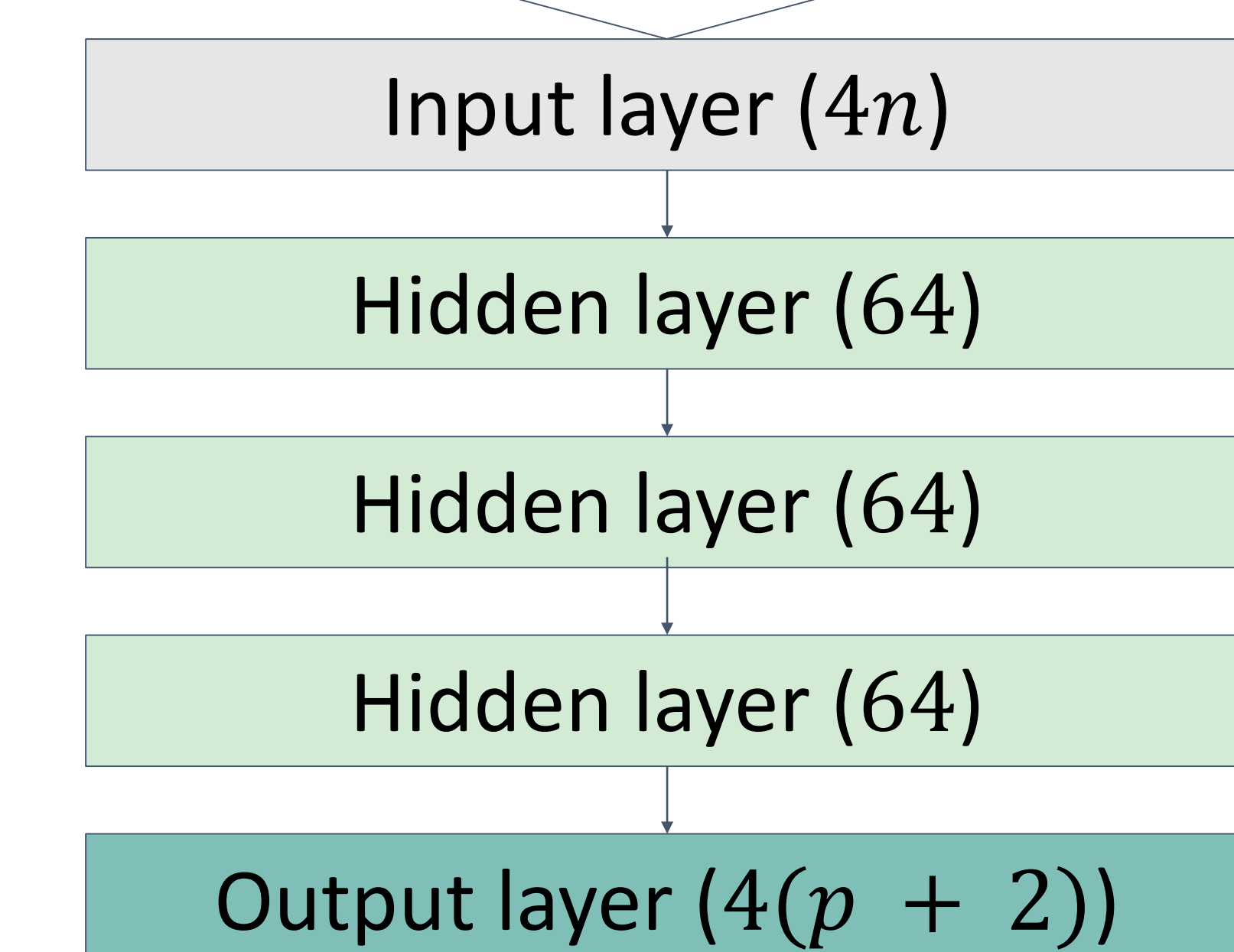
## Visualizing Estimations with Uncertainty



prior localizations, true +1s future localization (white), mean for +1s localization, location probability for +1s

## Method

$$\mathbf{B} = \{B_{-n+1}, B_{-n+2}, \dots, B_0\}$$



$$\theta_{\mathbf{B}} = \{\theta_B^x, \theta_B^y, \theta_B^w, \theta_B^h\}$$

$$\theta_{\sigma} = \{\theta_{\sigma}^x, \theta_{\sigma}^y, \theta_{\sigma}^w, \theta_{\sigma}^h\}$$

$p$ : polynomial degree

## Predicting Relative Transformations

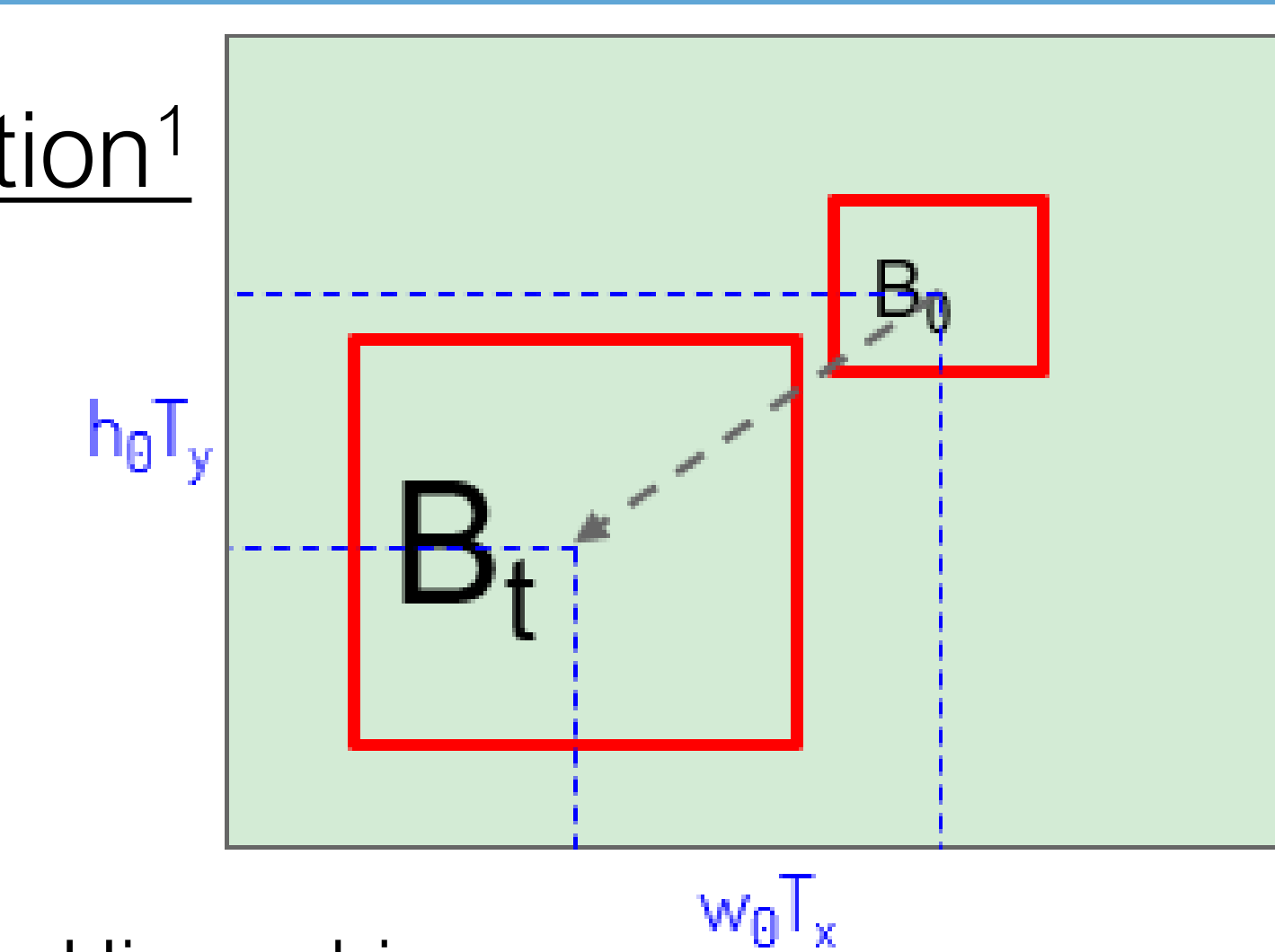
Scale-invariant transformation<sup>1</sup>

$$B_x(t) = w_0 T_x + x_0$$

$$B_y(t) = h_0 T_y + y_0$$

$$B_w(t) = w_0 \exp(Tw)$$

$$B_h(t) = h_0 \exp(Th)$$



<sup>1</sup>R. Girshick (2013), Rich Feature Hierarchies

## Model Output: Functions of Time

- Output is a set of coefficients for the bounding box predictor  $T_d$  and uncertainty model  $\sigma_d$
- Construct polynomial functions:

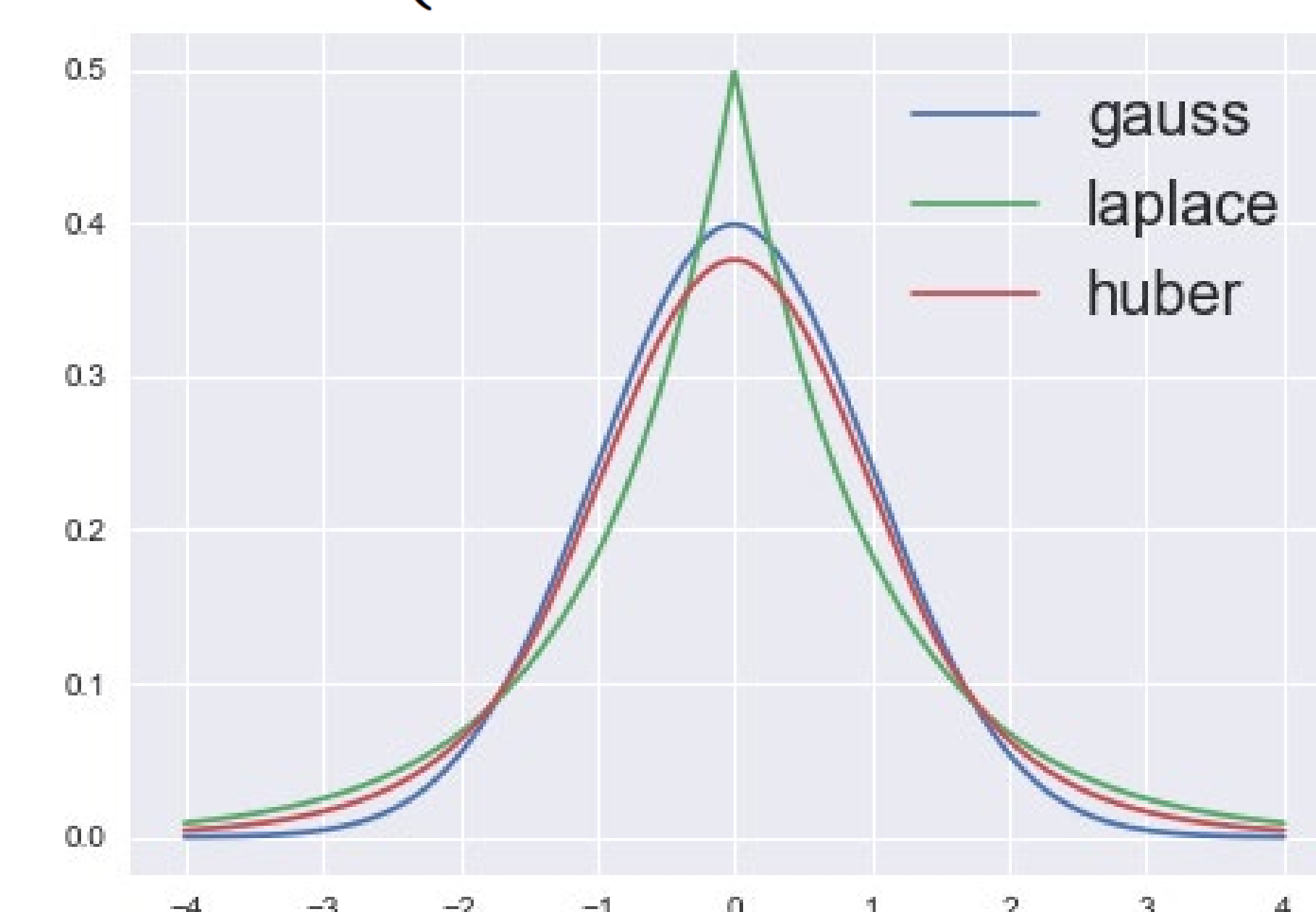
$$T_d(t; \theta_B^d) = \sum_{i=1}^p \theta_B^{d(i)} t^i.$$

$$\sigma_d(t; \theta_{\sigma}^d) = |\theta_{\sigma}^{d(1)} t| + |\theta_{\sigma}^{d(0)}|$$

## Localization Loss w/o Uncertainty

- Evaluate at 10 future timesteps
- Minimize error from predicted  $T^{(1)} \dots T^{(9)}$  to ground truth  $\hat{T}^{(1)} \dots \hat{T}^{(9)}$  using Huber loss

$$H(\hat{x}, x) = \begin{cases} \frac{1}{2} (\hat{x} - x)^2 & \text{if } |\hat{x} - x| < \tau \\ \tau |\hat{x} - x| - \frac{1}{2} \tau^2 & \text{otherwise,} \end{cases}$$



## Training for Confidence

- We maximize the likelihood of a distribution defined by the Huber loss
- Huber loss combines the  $L1$  and  $L2$  losses; the Huber distribution combines the *Gaussian* and *Laplace* distributions

$$p(\hat{x}|x, \sigma) = \begin{cases} \frac{1}{c} \exp\left(-\frac{(\hat{x}-x)^2}{2\sigma^2}\right) & \text{if } |\hat{x}-x| < \tau \\ \frac{1}{c} \exp\left(-\frac{\tau}{\sigma^2}|\hat{x}-x| + \frac{\tau^2}{2\sigma^2}\right) & \text{otherwise,} \end{cases}$$

### Objective function:

$$\min_{\theta_B^d, \theta_{\sigma}^d} \sum_d H_d(\hat{T}_d(t), T_d(t; \theta_B^d), \sigma_d(t; \theta_{\sigma}^d))$$

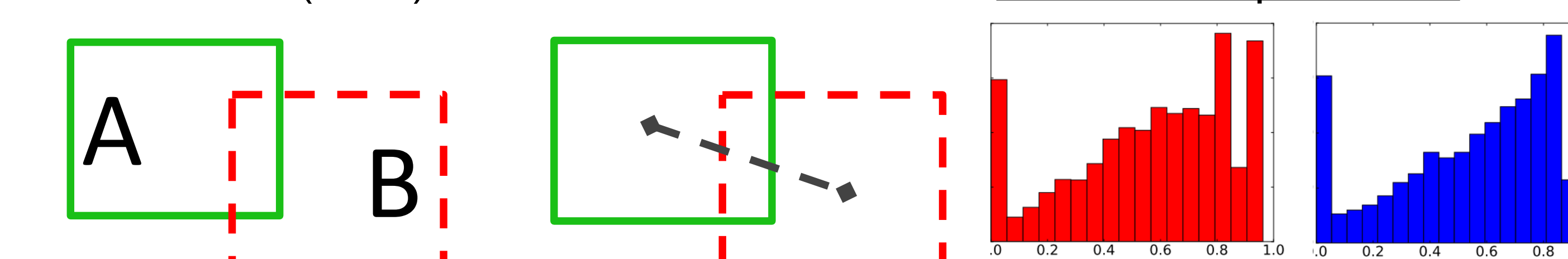
$$H_d(\hat{T}, T, \sigma) = \log c + \begin{cases} \frac{(\hat{T}-T)^2}{2\sigma^2} & \text{if } |\hat{T}-T| < \tau \\ \frac{\tau}{\sigma^2} |\hat{T}-T| - \frac{\tau^2}{2\sigma^2} & \text{otherwise,} \end{cases}$$

$$c = \sigma \sqrt{2\pi} \operatorname{erf}\left(\frac{\tau}{\sigma\sqrt{2}}\right) + \frac{2\sigma^2}{\tau} \exp\left(-\frac{\tau^2}{2\sigma^2}\right) \quad \tau = 1.345\sigma$$

## Results

### Comparison of Loss / Model Type

Intersection over Union (IoU)    Displacement Error    L1 vs. Huber IoU dist. comparison



Loss	Func.	DE		All ADE		IoU		Hard DE		IoU	
		+0.5s	+1.0s	+0.5s	+1.0s	+0.5s	+1.0s	+0.5s	+1.0s		
-	constant	32.06	72.15	36.98	0.498	0.339	44.23	102.40	51.62	0.326	0.128
-	linear	14.61	39.51	17.95	0.663	0.464	23.14	63.27	28.56	0.492	0.219
L1	$p=6$	12.81	29.05	14.78	0.697	0.564	17.06	38.06	19.56	0.607	0.475
L2	$p=6$	15.13	37.43	18.12	0.671	0.521	20.90	51.97	25.02	0.563	0.394
Huber	$p=6$	12.58	29.18	14.72	0.708	0.584	16.18	36.76	18.76	0.622	0.488
Huber	RNN	14.01	31.37	16.12	0.686	0.570	19.62	44.63	22.71	0.577	0.442

### Marginal log-probability distributions for +1s

Config.	L1	L2	Huber	Huber (RNN)
$\mathcal{H}^2(\mathcal{T}, \hat{\mathcal{T}})$	0.568	0.607	0.562	0.562

